

# Self assembling trees with RCCS

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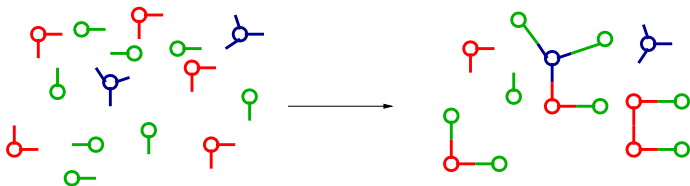
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# The problem

- **Question** : How a collective behaviour may emerge from elementary interactions (forward and backward)
- **Applications**
  - Molecular biology (backward)
  - Genetic engineering (forward)
  - Distributed robotics (forward)

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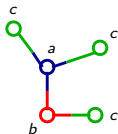


# A syntax for the trees

- $V$  : a set of nodes.
- $\delta : V \rightarrow \mathbb{N}$  : degree map.

$$(v, \{t_1, \dots, t_n\})$$

- $\text{edg}(t, v)$  : number of edges connected to  $v$  in  $t$
- $t$  is coherent  $\iff \forall v \in n(t), \delta(v) = \text{edg}(t, v)$



$$\delta(a) = 3 \quad \delta(b) = 2 \quad \delta(c) = 1$$

$$(a, \{(b, \{c\}), c, c\})$$

# An LTS for the specification

The **specification** :  $\text{SPEC} = (S, L, \rightarrow)$

- Element of the state space :  $(N, \sum_i t_i)$  with  $N \subseteq V$  and  $t_i$  coherent.
- Labels : set of coherent trees
- Transition relation : for all coherent tree  $t$

$$(N + n(t), \sum_i t_i) \rightarrow_t (N, t + \sum_i t_i)$$

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We are looking for an **implementation** in a process algebra:

- Concurrency
- Binary interactions
- Mathematical tool for proving correctness.

## Possible program

$\text{NODE}_i$	$\stackrel{\text{def}}{=} \tau.\text{BUILD}_{i^*}^{\delta(i),\emptyset} + \sum_{j \in I} r_{ij}.\text{BUILD}_{ij}^{\delta(i),\emptyset}$		
$\text{BUILD}_{ij}^{n+1,S}$	$\stackrel{\text{def}}{=} \sum_{k \in I} \bar{r}_{ik}.\text{BUILD}_{ij}^{n,S \cup \{k\}} + \text{kill}_i.\text{ABORT}_i^S$		
$\text{BUILD}_{i^*}^{n+1,S}$	$\stackrel{\text{def}}{=} \sum_{k \in I} \bar{r}_{ik}.\text{BUILD}_{ij}^{n,S \cup \{k\}} + \tau.\text{ABORT}_i^S$		
$\text{BUILD}_{i\alpha}^{0,S}$	$\stackrel{\text{def}}{=} \text{WAIT}_{i\alpha}^{ S ,S}$		
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$\text{WAIT}_{i^*}^{0,S}$	$\stackrel{\text{def}}{=} \text{ok}_i.\uparrow_{i^*}^S$		
$\text{FREE}^{S \cup \{i\}}(\text{end})$	$\stackrel{\text{def}}{=} \overline{\text{kill}_i}.\text{FREE}^S(\text{end})$	$\uparrow_{ij}^S \stackrel{\text{def}}{=} \tau.\uparrow_{ij}^S + \text{kill}_i.\text{ABORT}_i^S$	
$\text{FREE}^{\emptyset}(\text{end})$	$\stackrel{\text{def}}{=} \overline{\text{end}}.0$	$\uparrow_{i^*}^S \stackrel{\text{def}}{=} \tau.\uparrow_{i^*}^S$	
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# What is required ?

The code is very complicated:

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- more difficult to understand, reuse, patch, . . .

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The code is very complicated:

- equivalence with the specification
- more difficult to understand, reuse, patch, ...

Our approach:

- 1 forward code in CCS
- 2 good properties w.r.t specification
- 3 lift into RCCS (nothing to do)
- 4 application of a theorem: RCCS term is equivalent to the specification



# CCS : semantics

$$\frac{}{\sum_i \alpha_i \cdot p_i \rightarrow_{\alpha_i} p_i} \text{ (act)} \qquad \frac{p \rightarrow_{\alpha} p'}{p \parallel q \rightarrow_{\alpha} p' \parallel q} \text{ (par)}$$

$$\frac{p \rightarrow_{\bar{a}} p' \quad q \rightarrow_a q'}{p \parallel q \rightarrow_{\tau} p' \parallel q'} \text{ (synch)}$$

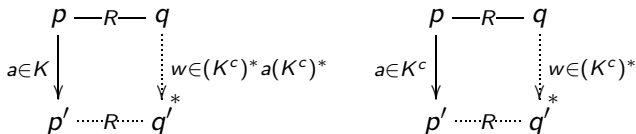
$$\frac{p \rightarrow_{\alpha} p' \quad \alpha \neq a, \bar{a}}{(a)p \rightarrow_{\alpha} (a)p'} \text{ (res)} \qquad \frac{p \equiv p' \rightarrow_{\alpha} q' \equiv q}{p \rightarrow_{\alpha} q} \text{ (equiv)}$$

$$D(\tilde{x}) \equiv p \text{ if } (D(\tilde{x}) := p) \in \Delta$$

# Mathematical tools

$K \subseteq A$ : *observable actions* (ex.  $K = A \setminus \{\tau\}$ )  
 $K^c := A \setminus K$ .

A relation  $R$  is a **simulation** if  $p R q$  implies:



It is also a **bisimulation** if symmetric.

# A solution ...

$$\text{NODE}_i \stackrel{\text{def}}{=} \tau.(\text{BUILD}_i^{\delta(i)} \mid \text{WAIT}_{i^*}^{\delta(i)}) \\ + \sum_{j \in V} r_{ij}.(\text{BUILD}_i^{\delta(i)-1} \mid \text{WAIT}_{ij}^{\delta(i)-1})$$

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$$\text{BUILD}_i^{n+1} \stackrel{\text{def}}{=} \sum_{j \in V} \bar{r}_{ij}.\text{BUILD}_i^n$$

$$\text{BUILD}_i^0 \stackrel{\text{def}}{=} 0$$



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$$\text{WAIT}_{i\alpha}^{n+1} \stackrel{\text{def}}{=} w_i.\text{WAIT}_{i\alpha}^n$$

$$\text{WAIT}_{ij}^0 \stackrel{\text{def}}{=} \bar{w}_j. \uparrow_j^i$$

$$\text{WAIT}_{i^*}^0 \stackrel{\text{def}}{=} \underline{\text{ok}}_i. \uparrow_{i^*}^i$$

## ...with deadlocks

If  $\delta(a) = 2$  and  $\delta(b) = \delta(c) = 1$

$$\begin{aligned}
 \text{NODE}_a \mid \text{NODE}_b \mid \text{NODE}_c &\rightarrow \text{BUILD}_a^2 \mid \text{WAIT}_{a^*}^2 \mid \text{NODE}_b \mid \text{NODE}_c \\
 &\rightarrow^* \text{WAIT}_{a^*}^2 \mid \text{WAIT}_{ba}^0 \mid \text{WAIT}_{ca}^0 \\
 &\equiv w_a \cdot w_a \cdot \underline{ok}_a \cdot \uparrow_{a^*}^a \mid \bar{w}_a \cdot \uparrow_a^b \mid \bar{w}_a \cdot \uparrow_a^c \\
 &\rightarrow^* \underline{ok}_a \cdot \uparrow_{a^*}^a \mid \uparrow_a^b \mid \uparrow_a^c \\
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 &\rightarrow^* \underline{\text{ok}_a} \cdot \uparrow_{a^*}^a \mid \uparrow_a^b \mid \uparrow_a^c \\
 &\rightarrow_{\text{ok}_a} \uparrow_{a^*}^a \mid \uparrow_a^b \mid \uparrow_a^c
 \end{aligned}$$

If  $\delta(a) = \delta(b) = 1$  and  $\delta(c) = 3$

$$\begin{aligned}
 \text{NODE}_a \mid \text{NODE}_b \mid \text{NODE}_c &\rightarrow \text{BUILD}_a^1 \mid \text{WAIT}_{a^*}^1 \mid \text{NODE}_b \mid \text{NODE}_c \\
 &\rightarrow \text{WAIT}_{a^*}^1 \mid \text{NODE}_b \mid \text{BUILD}_c^2 \mid \text{WAIT}_{ca}^2 \\
 &\rightarrow \text{WAIT}_{a^*}^1 \mid \text{WAIT}_{bc}^0 \mid \text{BUILD}_c^1 \mid \text{WAIT}_{ca}^2 \\
 &\equiv \text{WAIT}_{a^*}^1 \mid \bar{w}_c \cdot \uparrow_c^b \mid \text{BUILD}_c^1 \mid w_c \cdot w_c \cdot \bar{w}_a \cdot \uparrow_a^c \\
 &\rightarrow \text{WAIT}_{a^*}^1 \mid \uparrow_c^b \mid \text{BUILD}_c^1 \mid w_c \cdot \bar{w}_a \cdot \uparrow_a^c
 \end{aligned}$$

## Not enough ?

The implementation **is not bisimilar** to the specification  
But has some good properties:

- What is assembled is allowed
- It **may** find a way to simulate the specification

# RCCS : syntax

$$p, q ::= (p \parallel q) \mid \sum_i \alpha_i \cdot p_i \mid (a)p \mid D(\tilde{x}) \mid 0 \quad \text{CCS}$$

$r, s ::= m \triangleright p$	Threads
$\mid (r \parallel s)$	Parallel
$\mid (a)r$	Restriction

$m ::= \langle \theta, a, p \rangle \cdot m$	Synch
$\mid \langle \theta \rangle \cdot m$	Commit
$\mid \langle 1 \rangle \cdot m \mid \langle 2 \rangle \cdot m$	Fork address
$\mid \langle \rangle$	Empty

# RCCS : semantics

Transition:  $t = \langle r, \theta, \zeta, r' \rangle$   
 Labels:  $\theta \in \mathcal{I} \quad \zeta := \alpha \mid \alpha^-$

If  $\theta \notin \mathcal{I}(m)$ :

$$m \triangleright \alpha.p + q \xrightarrow{\theta:\alpha} \langle \theta, \alpha, q \rangle \cdot m \triangleright p \quad (\text{act})$$

$$\langle \theta, \alpha, q \rangle \cdot m \triangleright p \xrightarrow{\theta:\alpha^-} m \triangleright \alpha.p + q \quad (\text{act}^-)$$

$$m \triangleright \underline{\alpha}.p + q \xrightarrow{\theta:\alpha} \langle \theta \rangle \cdot m \triangleright p \quad (\underline{\text{act}})$$

## RCCS : semantics

Synchronisation rules are:

$$\begin{array}{c} \text{(com)} \frac{r \xrightarrow{\theta:\bar{a}} r' \quad s \xrightarrow{\theta:a} s'}{r \parallel s \xrightarrow{\theta:\tau} r' \parallel s'} \quad \frac{r \xrightarrow{\theta:\bar{a}^-} r' \quad s \xrightarrow{\theta:a^-} s'}{r \parallel s \xrightarrow{\theta:\tau^-} r' \parallel s'} \quad \text{(com}^-\text{)} \\ \\ \frac{r \xrightarrow{\theta:\bar{a}} r' \quad s \xrightarrow{\theta:a} s'}{r \parallel s \xrightarrow{\theta:\tau} r' \parallel s'} \quad \text{(com)} \end{array}$$

# RCCS : semantics

$$\begin{array}{l}
 m \triangleright (x)p \quad \equiv \quad (x)(m \triangleright p) \text{ if } x \notin m \\
 m \triangleright (p \parallel q) \quad \equiv \quad (\langle 1 \rangle \cdot m \triangleright p) \parallel (\langle 2 \rangle \cdot m \triangleright q)
 \end{array}$$

Context rules are:

$$\frac{r \xrightarrow{\theta:\zeta} r' \quad \theta \notin \mathcal{I}(s)}{r \parallel s \xrightarrow{\theta:\zeta} r' \parallel s} \text{ (par)} \quad \frac{r \xrightarrow{\theta:\zeta} r' \quad \zeta \neq x, \bar{x}, x^-, \bar{x}^-}{(x)r \xrightarrow{\theta:\zeta} (x)r'} \text{ (res)}$$

$$\frac{r \equiv r' \quad r' \xrightarrow{\theta:\zeta} s' \equiv s}{r \xrightarrow{\theta:\zeta} s} \text{ (equiv)}$$



# Causal traces

A trace  $\sigma$  is said to be causal if:

- 1 there is only one irreversible action  $t$
- 2 for all  $\sigma' \sim \sigma$ ,  $\sigma'$  ends with  $t$ .

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$$a.\underline{b}.0 \mid c.0 \xrightarrow{a} \underline{b}.0 \mid c.0 \xrightarrow{c} \underline{b}.0 \xrightarrow{\underline{b}} 0$$

$$a.\underline{b}.0 \mid c.0 \xrightarrow{a} \underline{b}.0 \mid c.0 \xrightarrow{\underline{b}} c.0$$

$$a.\underline{b}.0 \mid \bar{a}.0 \xrightarrow{\tau} \underline{b}.0 \xrightarrow{\underline{b}} 0$$

# Main theorem

Some definitions:

- $K$  a set of underlined action in CCS
- $p_1 \xrightarrow{\underline{k}}^c p_2$  if there is a causal trace from  $p_1$  to  $p_2$  ending with  $\underline{k} \in K$
- $CTS(p) = (P, p, K, \xrightarrow{c})$  : the causal transition system induced by  $p$

## Theorem

Let  $p$  be a CCS process and  $\Phi$  the relation  $\{(\underline{k}, \theta : \underline{k}) \mid \underline{k} \in K, \theta \in \mathcal{I}\}$ , then  $CTS(p) \approx_{\Phi} LTS(\langle \rangle \triangleright p)$ .

# On the implementation

What is not part of the theorem:

- mapping the trees into the CCS code:

$$\llbracket (a, \{t_1, \dots, t_n\}) \rrbracket_\alpha = \uparrow_\alpha^a \mid \llbracket t_1 \rrbracket_a \mid \dots \mid \llbracket t_n \rrbracket_a$$

- mapping the states of the LTS into the CCS code:

$$\llbracket N, \sum_i t_i \rrbracket = \prod_{i \in N} \text{NODE}_i \mid \prod_j \llbracket t_j \rrbracket_\star$$

## Proposition

Let  $\Phi$  be the relation  $\{(t, \underline{ok}_i) \mid i \in V\}$ .

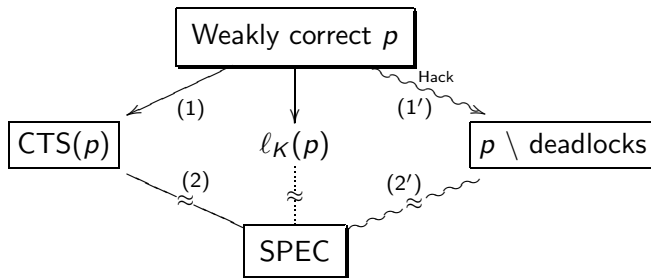
The relation  $\{(N, \sum_i t_i), \llbracket N, \sum_i t_i \rrbracket\}$  is a  $\Phi$ -bisimulation between SPEC and CTS( $\llbracket V \rrbracket$ ).

# Proving correctness in RCCS

Bisimulation is 1. and 2. *weak correctness* is 1. and 2'.

1. **Simulation:** All transactions of the spec. can be performed in the implementation.
2. **Correctness:** All evolutions of the implementation lead to a state which is also in the spec.
- 2'. **No bad state:** All evolutions of the implementation **that (causally) lead to a state** must be in accordance with the specification.

# Declarative Concurrent Programming



- (1) Automatic: Ocaml tool (Causal)
- (1') By hand. Difficulty depends on system's topology.
- (2) Automatic: in most cases  $CTS(p) \equiv SPEC$ .
- (2') By hand beyond a certain size

## Further works

Some drawbacks:

- General result may imply loss of efficiency when backtracking
- We strongly rely on the mapping  $[[\cdot]]$

Improvements:

- Having more refined labels (CCS with values)
- Dealing with graphs (Reversible  $\pi$ -calculus)
- Handling a stronger property on the correctness (stochastic behaviour)