The Information Bottleneck Method for Optimal Prediction of Multilevel Agent-based Systems

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1. General Setting: Information Bottleneck for Optimal Prediction

\[ \phi(X^t) \in S_a \]

\[ \psi(X^{t+\tau}) \in S_a \]

\[ \phi: \Sigma \rightarrow S_a \]

\[ \Sigma = S_a \]

\[ \tau \rightarrow \infty \]

\[ \tau = 0 \]

\[ \tau = 10 \]

\[ \tau = 20 \]

\[ \tau = 30 \]

\[ \tau = 40 \]

\[ \tau \rightarrow \infty \]

2. Observing Agent-based Systems with Additive Measurements

- Agent States: \( X_1 \in S, X_2 \in S, \ldots, X_N \in S \)
- System State: \( X^t = (X_1^{t}, X_2^{t}, \ldots, X_N^{t}) \in S^N \)

Definition: Additive Measurement

A family \( \mu(A; X) \) of measurements parametrized by an agent set \( A \subset \{1, \ldots, N\} \) such that:

- \( \mu(A; X) \) only depends on the state of agents in \( A \)

\[ \mu(A; X) = \Pr(\mu(A) | X) = \Pr(\mu(A) | (X)_a) \]

- from the observation of two disjoint agent sets, one can deduce the observation of their union:

\[ A_1 \cap A_2 = \emptyset \Rightarrow H(\mu(A_1; X) | \mu(A_2; X), A_1, A_2) = 0 \]

Properties of the Measurement Poset

- By combining several measurements, one refines the observation space:

\[ (\mu_1; \mu_2) \preceq (\mu_3; \mu_4) \]

- The MICRO measurement refines any other measurement:

\[ (\mu_1; \mu_2, \mu_3, \mu_4) \preceq (\mu_1, \mu_2, \mu_3, \mu_4) \]

- The EMPTY measurement is refined by any other measurement:

\[ (\mu_1; \mu_2, \mu_3, \mu_4) \preceq (\mu_1, \mu_2, \mu_3, \mu_4) \]

- Observing two nested agent sets is equivalent to observing the smaller set and its complement:

\[ A_1 \subset A_2 \Rightarrow \{ (\mu_1; \mu_2), \{ \mu_1, \mu_2 \} \} \preceq \{ (\mu_1, \mu_2), \mu_1, \mu_2 \} \]

3. The Measurement Poset

4. Application to the Voter Model: Predicting Synchronisation Processes in Interaction Graphs

- Agent States: \( X_1 \in [0,1], X_2 \in [0,1], \ldots, X_N \in [0,1] \)
- System State: \( X^t = (X_1^{t}, X_2^{t}, \ldots, X_N^{t}) \in [0,1]^N \)
- Transitions Kernel: \( T(X_1^{t+1} | X_i^{t-1}, X_j^{t-1}) \)
- Additive Measurements: \( A \subset \{1, \ldots, N\}, \mu(A; X^t) = \sum_{i \in A} X_i^t \)

Predicting MACRO in the Complete Graph (\( N = 7 \))

Predicting AGENT in the Complete Graph (\( N = 7 \))

Results:

- The predictive capacity of all measurements decreases as the horizon increases
- MICRO is more complex than MACRO
- MACRO is as predictive as MICRO
- Estimating the current state by sampling might provide more efficient prediction

Predicting AGENT in Two Communities (\( N = 10 + 10 \))

Results:

- From local to global dynamics: MICRO, MESO, and MACRO are respectively more efficient for short, middle, and long-term prediction
- Multilevel dynamics: These three measurements can be combined for higher predictive power depending on the horizon (including the three-level measurement MICRO + MESO + MACRO)

Predicting AGENT in the Ring (\( N = 9 \))

Results:

- The size of the neighbourhood to be observed depends on the prediction horizon (the higher the horizon is, the larger the observed neighbourhood should be)
- For a given prediction horizon, their is an optimal neighbourhood size that does not depend on the allowed complexity level

Bibliography